EFFECT OF ADDITIONAL VIBRATIONAL LOADING ON LIFETIME IN CONDITIONS OF ONE-DIMENSIONAL TENSION

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In the present paper, the effect of additional vibrational loading on the life of a sample subjected to tension is evaluated on the basis of the well-known ideas of S. N. Zhurkov. It is found that this effect may be pronounced, even for small amplitudes of the vibrations.

S. N. Zhurkov [1, 2], like other investigators (see, for example [3]), has introduced a kinetic concept of failure for the rupture of bonds at the molecular level. According to this concept, the life t^* of a sample subjected to tension, when the stresses σ and the temperature T vary during the loading process, is defined by the relation

$$\int_0^{t^*} \frac{dt}{\tau_0} \exp \frac{U - \gamma\sigma}{RT} = 1, \qquad \sigma_1 \leqslant \sigma_0 , \tag{1}$$

where τ_0, γ are material constants, U is the activation energy of the bond rupture reaction, and R is the universal gas constant.

Let the gas temperature be constant and

$$\sigma = \sigma_0 + \sigma_1 \sin \omega t , \tag{2}$$

where σ_0 is the principal tensile stress, σ_1 is the amplitude of the additional vibrational loading, and ω is a frequency such that $t^* \gg 2\pi/\omega$.

Substituting (2) into (1), after some transformations, we get

$$\int_0^1 \exp (a \sin 2\pi x)\, dx = \frac{t_0}{t^*}, \qquad a = \frac{\gamma\sigma_1}{RT} , \tag{3}$$

where t_0 is the lifetime under a constant tensile stress equal to σ_0. In conformity with [4], the integral in the left-hand side of (3) is equal to $I_0(a)$, where I_0 is a zero-order Bessel function of an imaginary argument.

If σ = const and the temperature varies according to the law

$$T = T_0 + T_1 \sin \omega t \qquad (T_1 \ll T_0) , \tag{4}$$

then it may be shown that making use of Frank-Kamenetskii's transformation [5] the relation for the lifetime retains the form (3), while the parameter a is defined by the relation

$$a = \frac{U - \gamma\sigma}{RT_0^2} T_1 . \tag{5}$$

Let the sample be subjected to a stress that varies according to the law (2), at a temperature that fluctuates in accordance with expression (4). Substituting (2) and (4) into (1) and considering only the terms of first order of smallness in the expansion of the exponential function in series in σ_1/σ_0 and T_1/T_0, we find that the lifetime is expressed in the form of (3), also in this case, while the quantity a is defined by the expression

$$a = \frac{\gamma\sigma_1}{RT_0} + \frac{U - \gamma\sigma_0}{RT_0^2} T_1 . \tag{6}$$

Let us obtain some estimates for a nylon fiber with the constants U = 45 kcal/mole, γ = 0.29 kcal/mole-mm^2/kg, for which, according to [2], $t_0 = 10^6$ sec at T = 300° and σ = 62 kg/mm^2. The additional vibrational load σ_1 = 5 kg/mm^2—which is less than 10% of the basic load—yields, in correspondence with (3), the value a = 2.4, which implies a change in lifetime by a factor of $I_0(2.4) = 3$.

For a described by the expression (5), a fluctuation of the temperature by 10° at $\sigma = \sigma_0$ leads to a change in lifetime by a factor of $I_0(1.5) = 1.7$. Under the simultaneous effect of these stress and temperature variations, the lifetime will decrease by a factor of $I_0(3.9) = 10.9$. It can be seen from the computation that even a small additional vibrational loading will appreciably reduce the lifetime of a sample. We wish to emphasize that these estimates do not include the effect of stress raisers, such as cracks, etc.

The results obtained are in good qualitative agreement with the experiments described in [3]; the presence of an additional vibratory load does not affect the exponential dependence of the lifetime on σ_0 but does decrease the lifetime. In [3], the magnitude of the additional vibratory loading is not given, so that a quantitative comparison between the analytical and experimental data is not possible.

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